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Landau damping and the onset of particle trapping in quantum plasmas

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Using analytical theory and simulations, we assess the impact of quantum effects on non-linear wave-particle interactions in quantum plasmas. We more specifically focus on the resonant interaction between Langmuir waves and electrons, which, in classical plasmas, lead to particle trapping. Two regimes are identified depending on the difference between the time scale of oscillation $t_B(k) = \sqrt{m/eEk}$ of a trapped electron and the quantum time scale $t_q(k) = 2m/\hbar k^2$ related to recoil effect, where E and k are the wave amplitude and wave vector. In the classical-like regime, $t_B(k) < t_q(k)$, resonant electrons are trapped in the wave troughs and greatly affect the evolution of the system long before the wave has had time to Landau damp by a large amount according to linear theory. In the quantum regime, $t_B(k) > t_q(k)$, particle trapping is hampered by the finite recoil imparted to resonant electrons in their interactions with plasmons.
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The notion of wave-particle interactions, the couplings between collective and individual particle behaviors, is fundamental to our comprehension of plasma phenomenology.[1–4] Landau damping, the collisionless damping of an electron plasma (Langmuir) wave, is the paradigmatic illustration of these interactions.[5] Resonant electrons with velocities sufficiently close to the wave phase velocity experience a nearly constant force and so can effectively exchange energy with the wave. For waves of small amplitude about a uniform equilibrium, the energy transfers overall result in the exponential decay of the wave amplitude in time at a rate $\gamma(k)$, where k is the wave number. In general, this linear theory prediction breaks down after a time $O(t_B(k))$ beyond which particles can get trapped and oscillate at a bounce frequency $\omega_B(k) = 1/t_B(k)$ in the wave troughs. Landau damping is effective provided $\gamma(k) \gg \omega_B(k)$, whereas when $\gamma(k) \ll \omega_B(k)$, the damping saturates and the wave amplitude remains finite (neglecting collisions).[6,7]

The question arises as to how wave-particle interactions are modified when the quantum nature of the electrons can no longer be ignored. Such is the case when the electrons' thermal energy k_BT is of the order of or smaller than their Fermi energy $E_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{1/3}$ (n and m are the electron density and mass). The physics of quantum plasmas (e.g., of the warm dense matter regime) is a frontier of high-energy density physics with relevance to many laboratory experiments and to astrophysics.[8] This field of research is being emboldened by new experimental facilities and high-performance computing. Nonlinear effects such as wave-particle couplings have been ignored and studies rely on linear response theory to model the experimental measurements (e.g., the X-ray Thomson scattering cross section[9]). In view of their significance in traditional plasmas, it is compelling to consider the nature and the role that wave-particle couplings take on in quantum plasmas. Few studies of non-linear effects in the quantum regime have been reported.[10] Of particular significance is the work by Suh *et al.*[11] who report simulations of the non-linear Landau damping using the one-dimensional quantum Liouville-Poisson system; they found that quantum effects can disrupt the hole formation in phase-space that, in the classical case, coincides with that saturation of Landau damping due to particle trapping. In this paper, we examine the onset of resonant particle trapping in small amplitude plasma waves across quantum degeneracy regimes.

To discriminate between quantum mechanical and quantum statistical effects, we systematically report the results for three models of electrons: quantum (q), semi-classical (sc), and classical (c) (a subscript is sometimes used to indicate a result pertaining to a specific model, e.g., $A =_{sc} \mathrm{B}$). The classical description is commonly used in plasma physics, where both dynamics and statistics are classical. For semi-classical electrons, the dynamics is classical (wave-mechanical effects like diffraction are overlooked), but the fermionic (Fermi-Dirac) statistics substitutes the Boltzmann statistics. The quantum description includes both quantum statistics and quantum mechanics by ascribing wave-like attributes to the electrons. In each case, we assume that ions do not participate in the high-frequency plasma oscillations and act as a uniform neutralizing background. Moreover, collisions between individual electrons are omitted from the analysis[12] and the plasma is described within the collective, mean-field approximation. We denote by $\omega_p = \sqrt{e^2 n/\epsilon_0 m}$ the electron plasma frequency, $a = (3/4\pi n)^{1/3}$ the Wigner-Seitz radius, $r_s == a/a_B = 1$ (a_B Bohr radius) the reduced density, $\theta = k_BT/E_F$ the degeneracy parameter, $f_{FD}(\mathbf{p}) = 1/[1 + \exp((\mathbf{p}^2/2m - \mu)/k_BT)]$ the Fermi-Dirac distribution, and $f_{MB}(\mathbf{p}) = n(k_BT/2\pi m)^{3/2} e^{-\mathbf{p}^2/2mk_BT}$ the Maxwell-Boltzmann distribution.

First, we evaluate the quantum effects on the basis of the usual trapping threshold criterion $\gamma(k) = \omega_B(k)$ for resonant electrons in a plasma wave created by an initial density perturbation $\delta n(\mathbf{r})/n = \epsilon \cos(kx)$. In the linear regime, the wave electric field $\mathbf{E}(\mathbf{r}, t) = E_0 e^{-\gamma(k)t} \sin(kx - \omega(k)t)\hat{x}$. Here, $E_0 = -en\epsilon/\epsilon_0 k$, and $\omega(k)$ and $\gamma(k)$ satisfy the dispersion equation

$$\epsilon(k,\omega(k)-i\gamma(k))=0\,, \tag{1}$$

where $\epsilon(k,z)=1-\frac{e^2}{\epsilon_0 k^2}\chi^0(k,z)$ is the analytic continuation of retarded dielectric function in the random-phase approximation (RPA), and χ^0 is the free-electron density response function[13,14]

$$\chi^0(k,z) =_q -\int d\mathbf{p}\frac{f_0(\mathbf{p}+\hbar\mathbf{k}/2)-f_0(\mathbf{p}-\hbar\mathbf{k}/2)}{\hbar z-\hbar\mathbf{k}\cdot\mathbf{p}/m} \tag{2}$$

$$=_{sc,c} -\int d\mathbf{p}\frac{\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{p}}f_0(\mathbf{p})}{z-\mathbf{k}\cdot\mathbf{p}/m}, \tag{3}$$

with $f_0 =_{q,sc} f_{FD} =_c f_{MB}$. The bounce frequency is $\omega_B(k)=\sqrt{-eE_0k/m}=\omega_p\sqrt{\epsilon}$.[6] To determine $\gamma(k)$, we solve *numerically* the dispersion relation for each electron model. The results for $\omega(k)$ and $\gamma(k)$ for $r_s=1$ and several degeneracy parameters θ across degeneracy regimes are shown in Fig. 1 as a function of the dimensionless wave-vector k/k_{sc},

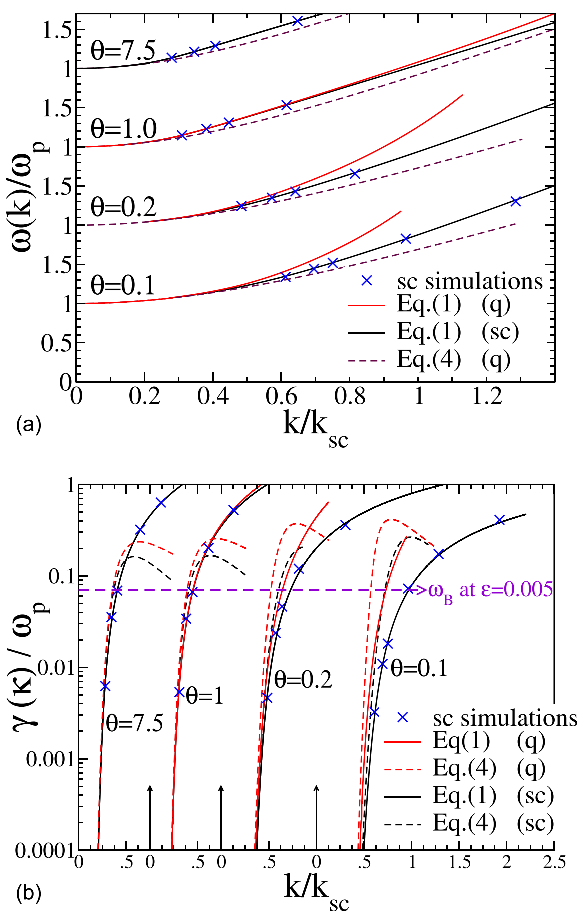

FIG. 1. Plasma wave dispersion relation (left) and linear Landau damping rates (right) for reduced density $r_s=1$ and degeneracy parameters between $\theta=7.5$ (nearly classical) to $\theta=0.1$ (strongly degenerate), obtained by numerically solving Eq. (1) for quantum (red lines) and semiclassical (black). In both figures, the dashed lines corresponds to the Bohm-Gross limit (4) for quantum electrons, the blue crosses to numerical semi-classical PIC simulations, and the curves have been shifted vertically (left) and horizontally (right) for clarity (in the right figure, a vertical arrow indicates the origin of the horizontal axis). For $\theta=7.5$, the quantum, semi-classical, and classical (not shown) results are indistinguishable to within the thickness of the curve.

where $k_s=-e^2\chi^0(0,0)/\epsilon_0$ is the inverse screening length. Also shown in Fig. 1 is the long wavelength (Bohm-Gross) limit

$$\omega^2(k)=\omega_p^2\left(1+\alpha^2k^2+o(k^2)\right), \quad \gamma(k)=\frac{\mathrm{Im}\,\epsilon(k,\omega(k))}{\frac{\partial\,\mathrm{Re}\,\epsilon}{\partial\omega}(k,\omega(k))}, \tag{4}$$

where $\alpha^2 =_{q,sc} 2\frac{I_{3/2}(\mu)}{I_{1/2}(\mu)}\frac{k_BT}{m} =_c 3\frac{k_BT}{m}$, where $I_\alpha(y)=\int_0^\infty\frac{x^\alpha}{1+\exp(x-y)}dx$ is the Fermi integral. We see that Landau damping is less and less effective as one penetrates the quantum regime: for a given wave number k, $\gamma_q(k)<\gamma_{sc}(k)<\gamma_{cl}(k)$ and the gap between these rates increases with decreasing θ. Thus, in light of the usual trapping criterion $\gamma(k)\ll\omega_B(k)$, one expects that resonant electrons in quantum plasmas are more readily prone to trapping than in classical plasmas. Moreover, Fig. 1(b) shows that, while the quantum and semiclassical descriptions are equal at small k, they markedly differ at intermediate k: this suggests that the electron wave-like character significantly modifies the nature of traditional wave-particle interactions.

To support the previous analysis, we perform numerical simulations for semi-classical electrons across the quantum degeneracy regimes. Like in traditional plasma physics, electrons are modeled by a phase-space distribution $f(\mathbf{r},\mathbf{p},t)$ satisfying the Vlasov-Poisson equations

$$\frac{\partial f}{\partial t}=-\frac{\mathbf{p}}{m}\cdot\nabla_{\mathbf{r}}f+\nabla_{\mathbf{r}}v\cdot\nabla_{\mathbf{p}}f, \tag{5a}$$

$$\nabla^2v(\mathbf{r},t)=\frac{e}{\epsilon_0}\left(\int d\mathbf{p}f(\mathbf{r},\mathbf{p},t)-n\right). \tag{5b}$$

But unlike in plasma physics, we use an initial condition consistent with the Pauli principle, namely,

$$f(\mathbf{r},\mathbf{p},t=0)=(1+\epsilon\cos(kx))f_{FD}(\mathbf{p})\,, \tag{6}$$

which describes a sinusoidal density perturbation (wave vector $\mathbf{k}=k\hat{x}$, amplitude ϵ) around the homogenous Fermi-Dirac equilibrium.[15] Note that the fermionic character is preserved since the Vlasov dynamics (5a) conserves phase-space volumes. Our numerical solution uses particle-in-cell techniques with parameters carefully chosen to ensure energy and entropy conservation (and hence the fermionic character), and to alleviate spurious noise due to particle discreteness.[16] Figure 2 shows the results of simulations for the time evolution of the Fourier component of the electric field $E(\mathbf{k},t)=-ik\tilde{v}(\mathbf{k},t)$ obtained for $r_s=1$, $\epsilon=0.005$, $\theta=0.01$, $\theta=0.1$ (degenerate), and 7.5 (classical regime). Results are shown for several wave-vectors k around the threshold value $k^*(\theta)$ defined such as $\gamma_{sc}(k^*)=\omega_B(k^*)$ and below which the usual trapping criterion $\gamma<\omega_B$ is satisfied; in Fig. 1, k^* is determined by the intersection of the horizontal dashed line with the black line, giving $k^*(\theta)/k_s=0.4, 0.97$ and 2.9 for $\theta=7.5$, 0.1, and 0.01. For $k>k^*$, after a fast transient time due to phase mixing, the electric field oscillates and its envelope decays exponentially in time with frequency and damping time in excellent agreement with linear theory (see blue crosses in Fig. 1). In contrast, when $k < k^*$, the wave amplitude displays a preliminary exponential Landau decay (again in remarkable accordance with linear theory as shown in Fig. 1), but then the latter saturates: the amplitude starts oscillating on a small frequency time scale, the amplitude of which increases with decreasing k. For $k \ll k^*$, in the time asymptotic limit, the oscillation in the electric field envelope disappears and the wave goes on propagating at nearly constant amplitude (e.g., see Fig. 2 for $\theta = 0.01$). These are signatures of trapping:[6,17] nonlinear effects stop the damping at a threshold in agreement with the theory, even for small amplitude waves, long before the wave has had a chance to damp by a substantial amount according to linear theory.

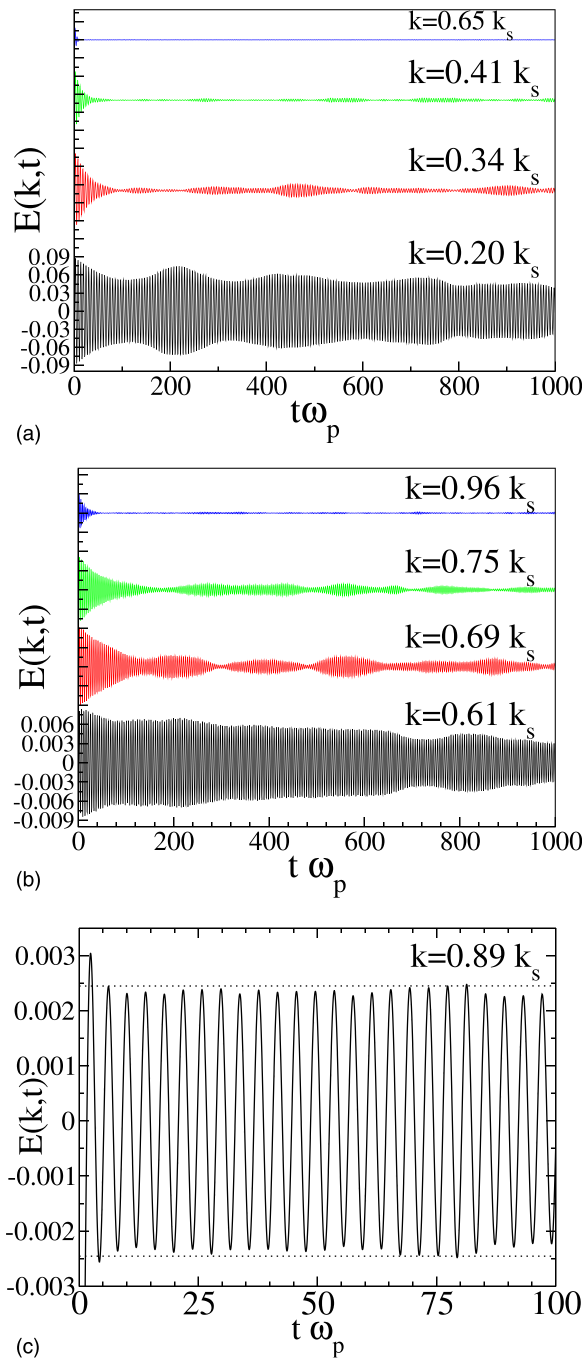

FIG. 2. Temporal evolution of the self-consistent electric field $E(\mathbf{k}, t)$ for semi-classical plasmas at reduced density $r_s = 1$ and degeneracy $\theta = 7.5$ (left), $\theta = 0.1$ (middle), and $\theta = 0.01$ (right), following an initial density perturbation $n(\mathbf{r}, t)/n = 0.005\cos(kx)$ at wave-vectors k distributed around the trapping threshold value $k^*(\theta)$ (see Fig. 1). Note that over the time duration of the plot for $\theta = 7.5$ and 0.1, the plasma oscillations are undistinguishable. Measured initial Landau decay rates and wave frequencies shown in Fig. 1 (blue crosses) are in excellent agreement with the dispersion relation results.

We now investigate whether quantum mechanics affects the usual trapping criterion. To facilitate the comparison between the quantum and classical descriptions, we work with the Wigner representation in which the system's state is described by a phase-space function $f(\mathbf{r}, \mathbf{p}, t)$, which, in many ways, resembles the classical distribution:[10,18] for example, the local particle density $n = \int d\mathbf{p}\, f$, momentum density $\mathbf{P} = \int d\mathbf{p}\mathbf{p}\, f$, and kinetic energy density $K = \int d\mathbf{p} \frac{\mathbf{p}^2}{2m} f$. In the mean-field approximation

$$\frac{\partial f}{\partial t} = -\frac{\mathbf{p}}{m}\cdot\frac{\partial f}{\partial \mathbf{r}} + I[f], \tag{7}$$

where

$$I[f] =_q \frac{i}{\hbar}\int d\mathbf{p}' \int \frac{d\mathbf{y}}{(2\pi)^3} e^{-i\mathbf{y}\cdot\mathbf{p}'} f(\mathbf{r}, \mathbf{p}+\mathbf{p}', t) \times [v(\mathbf{r}+\hbar\mathbf{y}/2) - v(\mathbf{r}-\hbar\mathbf{y}/2)] \tag{8a}$$

$$=_{sc,c} \frac{\partial v}{\partial \mathbf{r}}\cdot\frac{\partial f}{\partial \mathbf{p}} \quad (\text{cf.Eq.(5a)}), \tag{8b}$$

where v satisfies Eq. (5b).[19] Following O'Neil,[6] we determine the onset of particle trapping by looking at the breakdown of the linear approximation to the theory Eq. (8). We consider the case in which a monochromatic plasma wave is excited with $\mathbf{E}(\mathbf{r}, t) = E(t)\sin(\mathbf{k}\cdot\mathbf{r} - \omega(k)t)\hat{x}$, as produced by the initial condition (6) with $f_0 =_{q,sc} f_{FD} =_c f_{MB}$. To make analytical predictions, we assume that the wave amplitude remains constant on time scale $t < 1/\gamma(k)$, i.e., $E(t) = E_0$ for $t < 1/\gamma(k)$. Under these conditions, the linear solution of Eq. (7) is

$$f(\mathbf{r}, \mathbf{p}, t) = [1 + \epsilon\cos(\mathbf{k}\cdot(\mathbf{r} - \mathbf{p}\,t/m))] f_0(\mathbf{p}) + \frac{eE_0}{k}\frac{\cos(\mathbf{k}\cdot\mathbf{r} - \omega(k)t) - \cos\left(\mathbf{k}\cdot\mathbf{r} - \frac{\mathbf{k}\cdot\mathbf{p}}{m}t\right)}{\omega(k) - \frac{\mathbf{k}\cdot\mathbf{p}}{m}} \times \begin{cases} \frac{1}{\hbar}\left[f_0\left(\mathbf{p} + \frac{\hbar\mathbf{k}}{2}\right) - f_0\left(\mathbf{p} - \frac{\hbar\mathbf{k}}{2}\right)\right] & (\mathrm{q}) \\ \mathbf{k}\cdot\frac{\partial f_0(\mathbf{p})}{\partial \mathbf{p}} & (\mathrm{sc,\ c}) \end{cases} \tag{9}$$

with $f_0 =_{q,sc} f_{FD} =_c f_{MB}$; the first term describes the free-streaming of the initial perturbation, while the second term originates from the electron-wave couplings. For momenta such that $\frac{\mathbf{k}\cdot\mathbf{p}}{m} = \omega(k)$, the linear solution (9) exhibits a secularity proportional to t and we find

$$\frac{I[f]-I[f_0]}{I[f_0]}(\mathbf{r},\mathbf{p}=,t)=_q \sin(\mathbf{k}\cdot\mathbf{r}-\omega t)\frac{f_0(\mathbf{p}+\hbar\mathbf{k})-2f_0(\mathbf{p})+f_0(\mathbf{p}-\hbar\mathbf{k})}{f_0(\mathbf{p}+\hbar\mathbf{k}/2)-f_0(\mathbf{p}-\hbar\mathbf{k}/2)}\frac{eE_0}{k}\frac{\sin(\hbar k^2 t/2m)}{\hbar^2k^2/2m} + \cos(\mathbf{k}\cdot\mathbf{r}-\omega t)\frac{f_0(\mathbf{p}+\hbar\mathbf{k})-f_0(\mathbf{p}-\hbar\mathbf{k})}{f_0(\mathbf{p}+\hbar\mathbf{k}/2)-f_0(\mathbf{p}-\hbar\mathbf{k}/2)}\frac{eE_0}{k}\left[\frac{\cos(\hbar k^2 t/2m)-1}{\hbar^2k^2/2m}\right] \tag{10a}$$

$$=_{sc,c}\sin(\mathbf{k}\cdot\mathbf{r}-\omega t)\frac{\partial^2 f_0/\partial^2 p_x}{\partial f_0/\partial p_x}eE_0 t+\cos(\mathbf{k}\cdot\mathbf{r}-\omega t)\frac{eE_0 k}{m}\frac{t^2}{2}\,. \tag{10b}$$

For classical electrons, Eq. (10b) exhibits the usual result:[6] as t approaches $t_B(k)=(m/ekE)^{1/2}$ (see second term proportional to t^2), $\delta I = I[f]-I[f_0]$ outgrows the linear term and the linear approximation breaks down. For semi-classical electrons, Eq. (10b) shows that the same result holds when quantum statistics alone is accounted for. The quantum result (10a), on the other hand, displays an additional time scale $t_q(k)=2m/\hbar k^2$. For $t_q(k) \gg t_B(k)$, i.e., for density amplitudes

$$\epsilon \gg \frac{(ak)^4}{12r_s} \equiv \epsilon^*(k,r_s) \propto k^4/n, \tag{11}$$

the quantum result (10a) reduces to the classical (10b) for times t of order $t_B(k)$: the linear approximation breaks down as t approaches $t_B(k)$. For $t_q(k) \ll t_B(k)$ or $\epsilon \ll \epsilon^*(k,r_s)$, however, the term within brackets limits the growth of δI and the linear approximation does not break down as t approaches $t_B(k)$: the usual trapping criterion does not hold. In fact, as we shall explain, the recoil phenomenon due to the interaction of resonant electrons with plasmons prevents particle trapping from happening.

To interpret physically the new time scale $t_q(k)$, we find it useful to go over from the wave description of plasma waves to the particle description in terms of plasmons.[20] The latter is not widely used in plasma physics, perhaps because typical electrostatic waves contain a huge number of these elementary excitations. Yet, as we argue here, plasmons provide an insightful alternative picture of Landau damping and particle trapping valid for both quantum and classical electrons. The notion of plasmons, first derived from the collective coordinate method of Bohm and Pines,[21] is readily recovered within our approach as follows. We calculate the rate of change in kinetic energy $\langle K\rangle$ and momentum $\langle \mathbf{P}\rangle$ per wavelength $2\pi/k$ from the linear solution (9); in the long-wavelength limit, we find[22]

$$\frac{d\langle K\rangle}{dt}=2\gamma(k)\langle\mathcal{E}\rangle, \quad \frac{d\langle\mathbf{P}\rangle}{dt}=2\gamma(k)\langle\mathcal{E}\rangle\frac{\mathbf{k}}{\omega(k)}, \tag{12}$$

where $\langle\mathcal{E}\rangle=\epsilon_0 E_0^2/4$ is the electrostatic wave energy per wavelength, and

$$\begin{aligned}\gamma(k) &=_q \frac{2\pi n k}{\hbar\omega_p^3}\left[g_{FD}\left(\frac{m\omega(k)}{k}-\frac{\hbar k}{2}\right)-g_{FD}\left(\frac{m\omega(k)}{k}+\frac{\hbar k}{2}\right)\right]\\ &=_{sc} -\frac{2\pi n k^2}{\omega_p^3}g'_{FD}\left(\frac{m\omega(k)}{k}\right)\\ &=_c -\frac{2\pi n k^2}{\omega_p^3}g'_{MB}\left(\frac{m\omega(k)}{k}\right),\end{aligned} \tag{13}$$

is the explicit expression of γ given in Eq. (4) with $g(p_x)=\int dp_y dp_z f(p_x,p_y,p_z)$. Moreover, energy conservation implies $d\langle K\rangle/dt=-d\langle\mathcal{E}\rangle/dt$. By defining the number of energy quanta $\mathcal{N}_k=\langle\mathcal{E}\rangle/\hbar\omega(k)$ in the wave, the previous rate equations become

$$\frac{d\langle K\rangle}{dt}=-\hbar\omega(k)\frac{d\mathcal{N}_k}{dt}, \quad \frac{d\langle\mathbf{P}\rangle}{dt}=-\hbar\mathbf{k}\frac{d\mathcal{N}_k}{dt}, \tag{14}$$

and

$$\frac{d\mathcal{N}_k}{dt}=-2\gamma(k)\mathcal{N}_k\,. \tag{15}$$

Thus, the quantum of plasma wave behaves as a quasi-particle, the plasmon: the increase or decrease in the wave energy by one quantum is accompanied by the absorption or emission of electron energy $\hbar\omega(k)$ and momentum $\hbar\mathbf{k}$. Equation (15) describes the stimulated absorption and emission of plasmons leading to the Landau damping of the plasma wave at the rate $\gamma(k)$ (the factor 2 in Eq. (15) arises because $\mathcal{N}_k$ is proportional to the square of the wave amplitude). Note that in Eq. (15), we have omitted the term describing the (spontaneous) plasmon emission by electrons in the form a wake behind them, which requires fast electrons because $\omega(k)/k$ is usually large, and is thus negligible at or near equilibrium.[23]

We now return to our purpose in light of the plasmon concept. From momentum and energy conservation, for an electron to absorb (+) or emit (−) a plasmon $(\hbar\mathbf{k},\hbar\omega(k))$, its momentum $\mathbf{p}$ must satisfy

$$p=\mathbf{p}\cdot\frac{\mathbf{k}}{k}=_q m\frac{\omega(k)}{k}\mp\frac{\hbar k}{2}=_{sc,c} m\frac{\omega(k)}{k}\,; \tag{16}$$

in the process, its energy changes by

$$\pm\hbar\omega(k)=_q \pm\hbar\mathbf{k}\cdot\frac{\mathbf{p}}{m}+\frac{\hbar^2\mathbf{k}^2}{2m}=_{sc,c}\pm\hbar\mathbf{k}\cdot\frac{\mathbf{p}}{m}\,. \tag{17}$$

This simple calculation reveals two important effects of the wave-like, diffractive nature of electrons. First, the resonant momentum condition is shifted by $\mp\hbar k/2$ with respect to the classical and semi-classical conditions; this in turn affects the relation of the Landau damping rate to the shape of the distribution function around $m\omega(k)/k$ as found in Eq. (13) and Fig. 1. Second, the energy transfer differs by the recoil energy $E_{\rm rec}(k)=\frac{\hbar^2\mathbf{k}^2}{2m}$ associated with the momentum transfer $\pm\hbar\mathbf{k}$. The quantum time-scale found earlier is intimately related to recoil since $t_q(k) = \hbar/E_{\rm rec}(k)$ and the two regimes delineated by ϵ^* can be understood as follows. In the wave frame, a trapped electron oscillates in a potential well of height $V_{trap}(k) = 2eE_0/k$ such that $\frac{V_{trap}(k)}{E_{\rm rec}(k)} = \left(\frac{t_q(k)}{t_B(k)}\right)^2$. Thus, for amplitudes ϵ smaller than $\epsilon^*(k, r_s)$, $V_{trap}(k) \leq E_{\rm rec}(k)$, and the recoil of a resonant electron that absorbs or emits a plasmon, kicks it outside the potential well: quantum diffraction lowers the trapping lifetime of resonant particles and deteriorates trapping overall. When Eq. (11) is satisfied, $V_{trap}(k) > E_{\rm rec}(k)$, most electrons can remain trapped by the wave and the usual theory remains qualitatively valid.

In summary, our study reveals two distinct regimes for the collisionless damping of a monochromatic plasma wave in quantum plasmas. First, a classical-like regime, where the nonlinear trapped-particle dynamics of resonant electrons, qualitatively akin to that well-known in classical plasmas, sets in long before the wave has had time to damp by a substantial amount. Second, a truly quantum regime, where particle trapping is hampered by the finite recoil imparted to resonant electrons in their interactions with the wave. Given the importance of wave-particle couplings in traditional plasmas, further work to comprehend them in quantum plasmas may be beneficial both from a fundamental physics standpoint and as a practical matter.

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[23]Accounting for spontaneous emission, Eq. (15) becomes $\frac{d\mathcal{N}_k}{dt} = 2\gamma(k)\mathcal{N}_k + \Gamma^{sp}(k)$, where $\Gamma_{sp}(k)$ is the rate of spontaneous plasmon emission. A traditional stopping-power calculation gives $\Gamma_{sp}(k) = \frac{e^2}{\epsilon_0 k^2}\omega(k)^2 \int d\mathbf{p} f(\mathbf{p})\delta\left(\omega(k) - \frac{\mathbf{k}\cdot\mathbf{p}}{m}\right) = \frac{e^2 m^3 \omega(k)^2}{k^3} g\left(\frac{\omega(k)}{k}\right)$, which is negligible for a thermal distributions.[13]